

# PERCENTAGES

## SIMPLE PERCENTAGE

(1) Invest £12000 for 8 months at 6% p.a.

for 1 year       $£12000 \div 100 \times 6 = £720$

for 8 months     $£720 \div 12 \times 8 = \underline{\underline{£480}}$

(2) A radio costs £60 excluding VAT at 20%.  
Find the cost including VAT.

VAT               $£60 \div 100 \times 20 = £12$

COST             $£60 + £12 = \underline{\underline{£72}}$

## EXPRESSING CHANGE AS A PERCENTAGE

$$\% \text{ CHANGE} = \frac{\text{CHANGE}}{\text{START VALUE}} \times 100\%$$

A £15000 car is resold for £12000.

actual loss =  $£15000 - £12000 = £3000$

$$\% \text{ loss} = \frac{\text{loss}}{\text{start value}} \times 100\%$$

$$= \frac{£3000}{£15000} \times 100\%$$

$$= 20\%$$

## COMPOUND PERCENTAGE

**INCREASE:** growth, appreciation, compound interest

**DECREASE:** decay, depreciation

$$\begin{array}{lcl} \text{original value} & & \text{changed value} \\ 100\% & \xrightarrow{+ a\%} & (100 + a)\% \\ 100\% & \xrightarrow{- a\%} & (100 - a)\% \end{array}$$

$$\begin{array}{lcl} 8\% \text{ growth} & 100\% \xrightarrow{+ 8\%} & 108\% \\ & & \times 1.08 \text{ to increase by } 8\% \end{array}$$

$$\begin{array}{lcl} 8\% \text{ decay} & 100\% \xrightarrow{- 8\%} & 92\% \\ & & \times 0.92 \text{ to decrease by } 8\% \end{array}$$

## APPRECIATION, DEPRECIATION

A £200000 house appreciated by 4% the first year, appreciated by 6% the second year and then depreciated by 5% the third year.

$$\pounds 200000 \times 1.04 \times 1.06 \times 0.95 = \underline{\underline{\pounds 209456}}$$

## COMPOUND INTEREST

The interest gained is added to the investment and interest is then paid on the increased investment.

Invest £80000 at 5% pa for 3 years.

$$£80000 \times (1.05)^3 = £92610$$

$$\text{Compound Interest } £92610 - £80000 = \underline{\underline{£12610}}$$

or year by year

$$\begin{aligned} \text{year 1 } & £80000 \div 100 \times 5 = £4000 \\ & £80000 + £4000 = £84000 \end{aligned}$$

$$\begin{aligned} \text{year 2 } & £84000 \div 100 \times 5 = £4200 \\ & £84000 + £4200 = £88200 \end{aligned}$$

$$\begin{aligned} \text{year 3 } & £88200 \div 100 \times 5 = £4410 \\ & £88200 + £4410 = £92610 \end{aligned}$$

$$\text{Compound Interest } £92610 - £80000 = \underline{\underline{£12610}}$$

## REVERSE PERCENTAGES

(1) Including 20% VAT a radio costs £96.

What is the cost excluding VAT?

increase value by 20%  
to £96

$$\begin{aligned} \text{value} \times 1.20 &= \text{£}96 \\ \text{value} &= \text{£}96 \div 1.20 \\ &= \underline{\underline{\text{£}80}} \end{aligned}$$

### NON-CALCULATOR

$$\begin{aligned} 120\% \div 12 &= 10\% \\ 10\% \times 10 &= 100\% \end{aligned}$$

$$\begin{aligned} \text{£}96 \div 12 &= \text{£}8 \\ \text{£}8 \times 10 &= \underline{\underline{\text{£}80}} \end{aligned}$$

(2) A camera costs £120 after a 25% discount.

What is the cost before the discount?

decrease value by 25%  
to £120

$$\begin{aligned} \text{value} \times 0.75 &= \text{£}120 \\ \text{value} &= \text{£}120 \div 0.75 \\ &= \underline{\underline{\text{£}160}} \end{aligned}$$

### NON-CALCULATOR

$$\begin{aligned} 75\% &= \frac{3}{4} \\ \frac{3}{4} \div 3 &= \frac{1}{4} \\ \frac{1}{4} \times 4 &= \frac{4}{4} \end{aligned}$$

$$\begin{aligned} \text{£}120 \div 3 &= \text{£}40 \\ \text{£}40 \times 4 &= \underline{\underline{\text{£}160}} \end{aligned}$$

# FRACTIONS

## MIXED NUMBERS

$$\begin{aligned}2\frac{3}{4} &= 2 + \frac{3}{4} \\ &= \frac{8}{4} + \frac{3}{4} \\ &= \frac{11}{4}\end{aligned}$$

$$2\frac{3}{4} = \frac{11}{4}$$

$4 \times 2 + 3 = 11$

$$\begin{aligned}\frac{11}{4} &= \frac{8}{4} + \frac{3}{4} \\ &= 2 + \frac{3}{4} \\ &= 2\frac{3}{4}\end{aligned}$$

$$\frac{11}{4} = 2\frac{3}{4}$$

$11 \div 4 = 2 \text{ R } 3$

EQUAL FRACTIONS eg.  $\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

SIMPLIFYING: eg.  $\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$

## ADD and SUBTRACT

Requires a common denominator - the **LCM**

(least common multiple)

For mixed numbers:

treat fractions and whole numbers separately.

$$\begin{aligned}&12\frac{5}{6} - 3\frac{2}{9} \\ &= 12\frac{15}{18} - 3\frac{4}{18} \\ &= 9\frac{11}{18}\end{aligned}$$

MULTIPLY  $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

"of" means  $\times$

change mixed numbers to "top-heavy" fractions

$$\begin{aligned} (1) \quad & \frac{3}{10} \text{ of } 2 \frac{3}{4} \\ & = \frac{3}{10} \times \frac{11}{4} \\ & = \frac{33}{40} \end{aligned}$$

$$\begin{aligned} (2) \quad & 1 \frac{2}{3} \times 3 \frac{1}{5} \\ & = \frac{5}{3} \times \frac{16}{5} \\ & = \frac{80}{15} \\ & = \frac{16}{3} \\ & = 5 \frac{1}{3} \end{aligned}$$

DIVIDE  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$   
RECIPROCAL

$$\begin{aligned} (1) \quad & \frac{5}{6} \div \frac{3}{7} \\ & = \frac{5}{6} \times \frac{7}{3} \\ & = \frac{35}{18} \\ & = 1 \frac{17}{18} \end{aligned}$$

$$\begin{aligned} (2) \quad & 1 \frac{5}{7} \div 4 \\ & = \frac{12}{7} \div \frac{4}{1} \\ & = \frac{12}{7} \times \frac{1}{4} \\ & = \frac{12}{28} \\ & = \frac{3}{7} \end{aligned}$$

## ORDER OF CALCULATION

$\times$  and  $\div$  before  $+$  or  $-$

to change the order use brackets :

**BRACKETS FIRST!**

$$(1) \quad \frac{5}{8} - \frac{2}{9} \times \frac{3}{4}$$

$$\begin{array}{l} \frac{2}{9} \times \frac{3}{4} \\ = \frac{6}{36} \\ = \frac{1}{6} \end{array} \qquad \begin{array}{l} \frac{5}{8} - \frac{1}{6} \\ = \frac{15}{24} - \frac{4}{24} \\ = \underline{\underline{\frac{11}{24}}} \end{array}$$

$$(2) \quad \left( \frac{5}{6} - \frac{1}{2} \right) \div 1\frac{3}{4}$$

$$\begin{array}{l} \frac{5}{6} - \frac{1}{2} \\ = \frac{5}{6} - \frac{3}{6} \\ = \frac{2}{6} \\ = \frac{1}{3} \end{array} \qquad \begin{array}{l} \frac{1}{3} \div 1\frac{3}{4} \\ = \frac{1}{3} \div \frac{7}{4} \\ = \frac{1}{3} \times \frac{4}{7} \\ = \underline{\underline{\frac{4}{21}}} \end{array}$$

# STATISTICS

## PROBABILITY

The probability of an event  $A$  occurring:

$$P(A) = \frac{\text{number of outcomes involving } A}{\text{number of outcomes possible}}$$

$$0 \leq P \leq 1$$

$$P = 0 \quad \text{impossible}$$

$$P(A) + P(\text{not } A) = 1$$

$$P = 1 \quad \text{certain}$$

Expected outcomes =  $P(A) \times$  number of trials

Roll two dice 36 times and hope to score a total of five.  
How many scores are expected not to be five ?

36 outcomes possible:

(1,1) (1,2) (1,3) ..... (1,6)  
(2,1) (2,2) (2,3) ..... (2,6)  
.  
.  
(6,1) (6,2) (6,3) ..... (6,6)

4 outcomes total five:

(1,4) (2,3) (3,2) (4,1)

$$P(\text{five}) = \frac{4}{36} = \frac{1}{9}$$

$$P(\text{not five}) = \frac{8}{9}$$

$$\text{number not five} = \frac{8}{9} \times 36 = 32$$



## STATISTICAL MEASURES

### FIVE FIGURE SUMMARY:

Splits the data so each section has  $\frac{1}{4}$  of the results.

$Q_1$  and  $Q_3$      the lower and upper quartiles

$Q_2$                  the median

L and H              the lowest and highest result

### MEASURES OF SPREAD:

**RANGE**

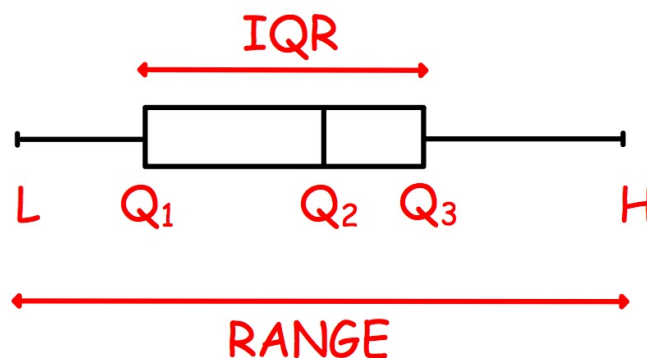
$$\text{RANGE} = H - L$$

**INTERQUARTILE RANGE**

$$\text{IQR} = Q_3 - Q_1$$

### BOX PLOT:

A statistical diagram showing the spread of the data, illustrating the information of the 5 figure summary.



19 results:

4, 6, 6, 8, 9, 10, 12, 13, 15, 16, 17, 17, 18, 18, 20, 22, 24, 25, 28

$L = 4$      $Q_1 = 9$              $Q_2 = 16$              $Q_3 = 20$              $H = 28$

$$\text{RANGE} = H - L$$

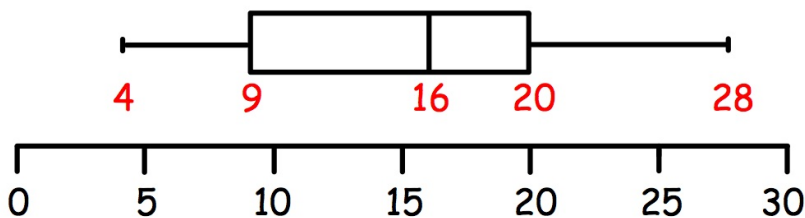
$$= 28 - 4$$

$$= 24$$

$$\text{IQR} = Q_3 - Q_1$$

$$= 20 - 9$$

$$= 11$$



for an imprecise  
scale write figures

Quartiles may lie between results:

18 results:

4, 6, 6, 8, 9, 10, 12, 13, 15, 16, 17, 17, 18, 18, 20, 22, 24, 25, 28

17 results:

4, 6, 6, 8, 9, 10, 12, 13, 15, 16, 17, 17, 18, 18, 20, 22, 24, 25, 28

16 results:

4, 6, 6, 8, 9, 10, 12, 13, 15, 16, 17, 17, 18, 18, 20, 22, 24, 25, 28

## COMPARING DATA

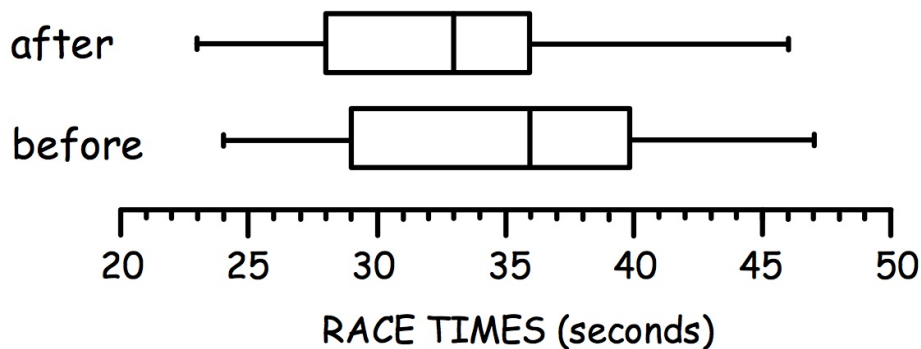
IQR measures spread (variation)

comment on:

(i) MEDIAN: lower/higher - is this good or bad ?

(ii) IQR: lower/higher - results less/more varied  
(consistent)

Army recruits given training:



Training has improved performance:

median time has reduced from 36 to 33

IQR reduced from 11 to 8 so times more consistent

## STANDARD DEVIATION

A measurement of how results deviate from the mean.

measures spread (variation) as a single number  
uses all the results

## COMPARING DATA

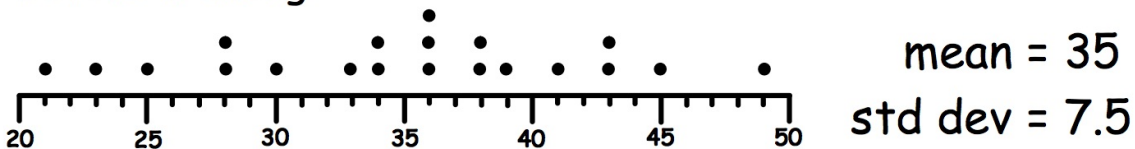
comment on:

(i) MEAN: lower/higher - is this good or bad ?

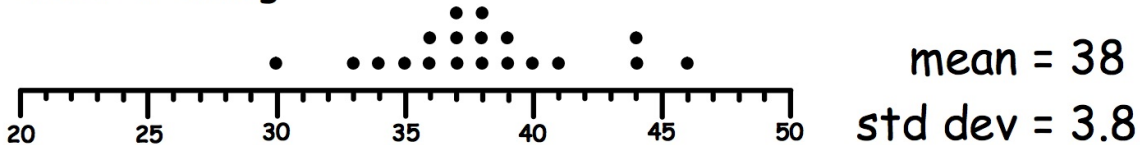
(ii) STD. DEV. : lower/higher - results less/more varied  
(consistent)

20 army recruits doing laps of a track in a fixed time.  
Comment on the effectiveness of the training.

before training



after training



Training has improved performance:

mean higher so they average more laps

std dev reduced so results more consistent

The pulse rate of 7 army recruits were recorded:  
69, 71, 73, 74, 76, 77 and 78 beats per minute.

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
69	-5	25
71	-3	9
73	-1	1
74	0	0
76	+2	4
77	+3	9
78	+4	16
518		64

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{518}{7}$$

$$= 74$$

$$\sum x = 518$$

$$\sum (x - \bar{x})^2 = 64$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{64}{6}}$$

$$= \sqrt{10.66\dots}$$

$$= 3.265\dots$$

$$\approx \underline{\underline{3.3}}$$

OR

$$\sum x = 69 + 71 + \dots + 78 = 518$$

$$\sum x^2 = 69^2 + 71^2 + \dots + 78^2 = 38396$$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$$

$$= \sqrt{\frac{38396 - \frac{(518)^2}{7}}{6}}$$

$$= \sqrt{\frac{64}{6}}$$

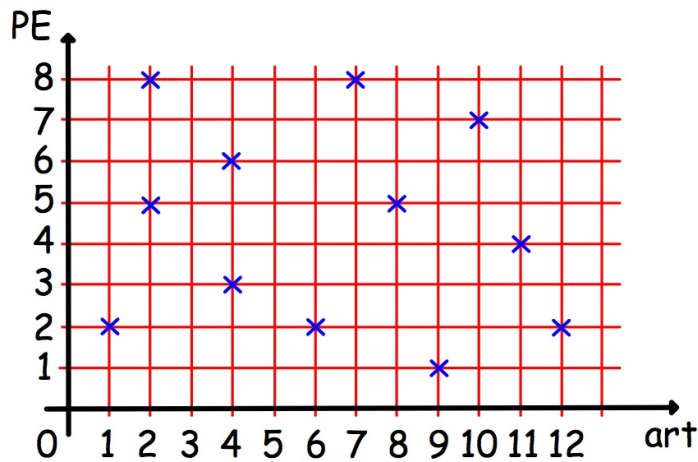
$$= 3.265\dots$$

$$\approx \underline{\underline{3.3}}$$

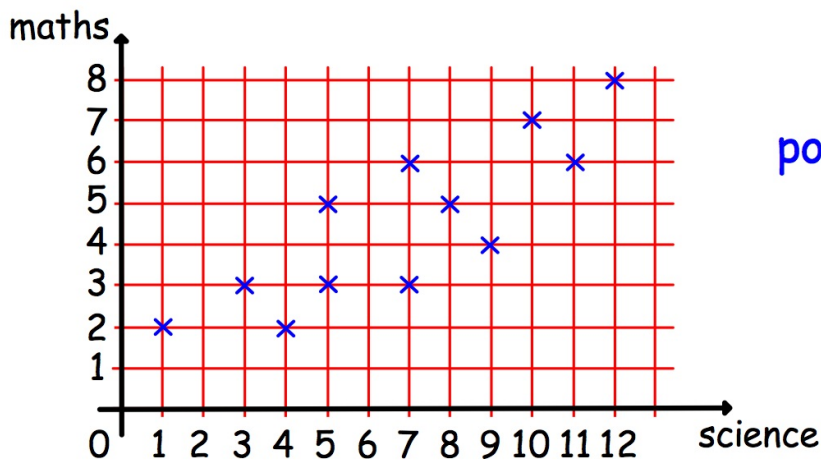
# SCATTER DIAGRAMS

If the points plotted lie along a line there is a relationship between the quantities.

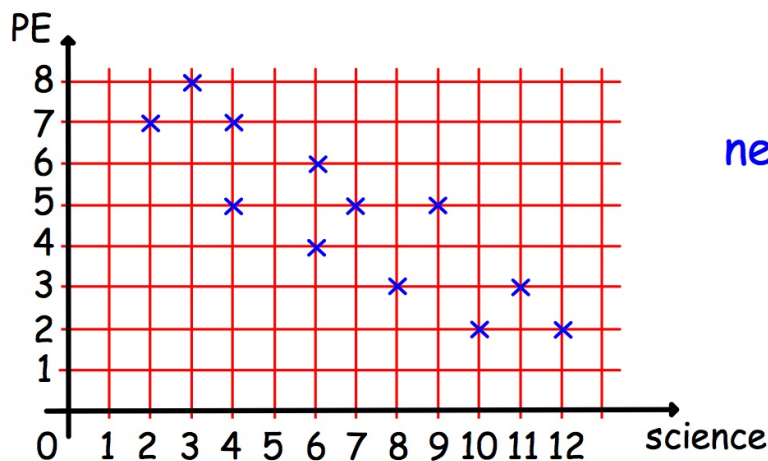
art	2	7	4	8	9	6	11	2	10	1	4	12
PE	8	8	3	5	1	2	4	5	7	2	6	2



no correlation



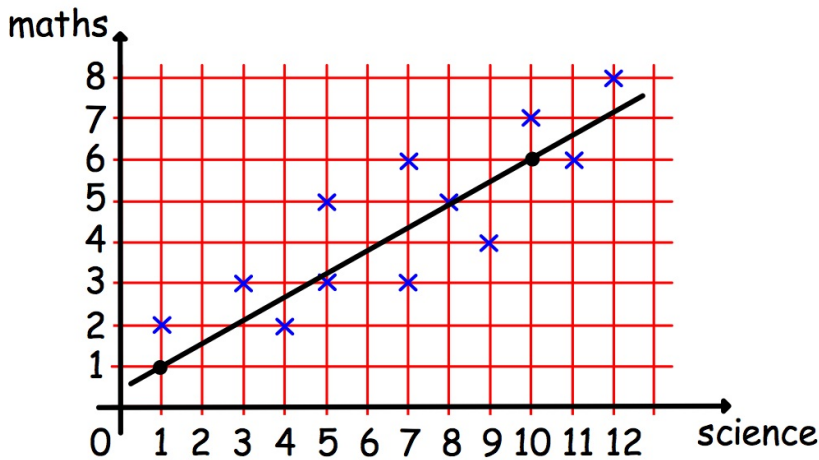
positive correlation



negative correlation

# LINE OF BEST FIT

Use two well-spaced points that lie on the line.



Find a formula connecting Maths and Science marks.

Equation of the line through (10,6) and (1,1).

$$\begin{array}{l}
 x_2 \quad y_2 \\
 (10, 6) \\
 \\
 x_1 \quad y_1 \\
 (1, 1)
 \end{array}
 \quad
 m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 1}{10 - 1} = \frac{5}{9}$$

$$y - b = m(x - a)$$

$$\begin{array}{l}
 a \quad b \\
 (1, 1) \\
 m = 5/9
 \end{array}$$

$$y - 1 = \frac{5}{9}(x - 1)$$

or can use (10,6)

$$y - \frac{9}{9} = \frac{5}{9}x - \frac{5}{9}$$

$$\underline{\underline{M = \frac{5}{9}S + \frac{4}{9}}}$$