NATIONAL 5 MATHEMATICS

COURSE NOTES

Applications

+ elements of N4

I

MATHS

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Sine rule: $\dfrac{\text{a}}{\text{sinA}} = \dfrac{\text{b}}{\text{sinB}} = \dfrac{\text{c}}{\text{sinC}}$

Cosine rule: $\text{a}^2 = \text{b}^2 + \text{c}^2 - 2bc\cos A$ or $\text{cosA} = \dfrac{\text{b}^2 + \text{c}^2 - \text{a}^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2}ab\sin C$

Volume of a sphere: Volume $= \frac{4}{3}\pi r^3$

Volume of a cone: Volume $= \frac{1}{3}\pi r^2 h$

Volume of a pyramid: Volume $= \frac{1}{3}Ah$

Volume of a cylinder: Volume $= \pi r^2 h$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

INDEX:

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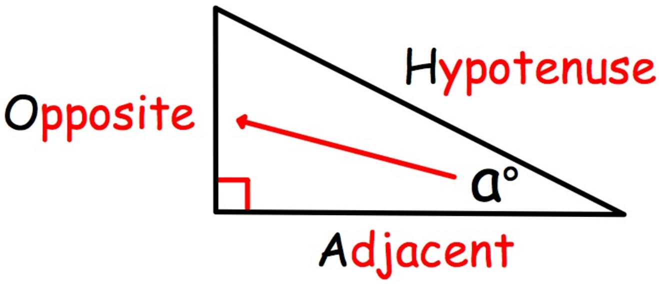
TRIGONOMETRY: FORMULAE

For RIGHT-ANGLED triangles,
the sine, cosine and tangent ratios are:



$\sin a° = {}^{O}/_{H}$

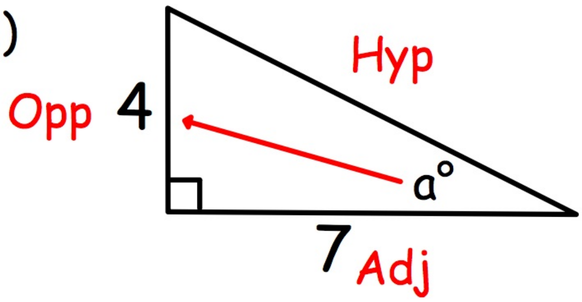
$\cos a° = {}^{A}/_{H}$

$\tan a° = {}^{O}/_{A}$

Memory Aid: SOH - CAH - TOA

FINDING AN ANGLE: need to know two sides

(1)



$\tan a° = {}^{4}/_{7}$

$a = \tan^{-1}({}^{4}/_{7})$

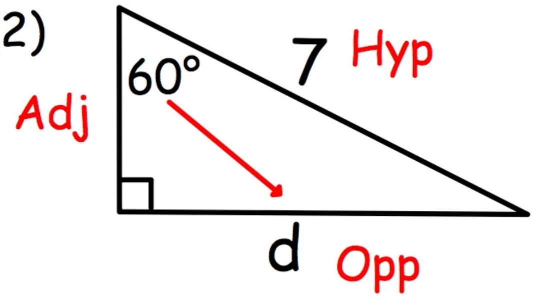
$= 29.744...$

$a \approx 29.7$

know O , know A SOH CAH TOA $\tan a° = {}^{O}/_{A}$

FINDING A SIDE: need to know an angle and a side

(2)



$\sin 60° = {}^{d}/_{7}$

$d = 7 \times \sin 60°$

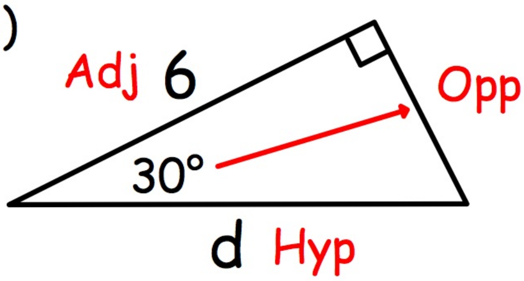
$= 6.062...$

$d \approx 6.1$

know H , find O   SOH CAH TOA $\sin a° = {}^{O}/_{H}$

with the letter as the denominator:

(3)



$\cos 30° = {}^{6}/_{d}$

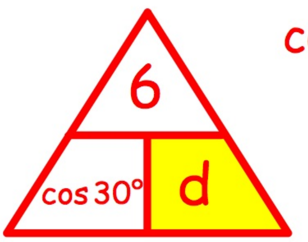
$d = {}^{6}/_{\cos 30°}$

$= 6.928...$

$d \approx 6.9$

know A , find H   SOH CAH TOA $\cos a° = {}^{A}/_{H}$

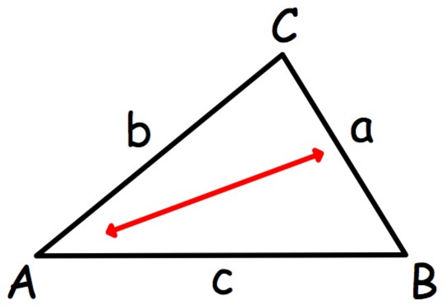
solving the equation when the letter is on the bottom



$$\cos 30° = \frac{6}{d}$$

$$d = \frac{6}{\cos 30°}$$

SINE RULE must know one side and its opposite angle

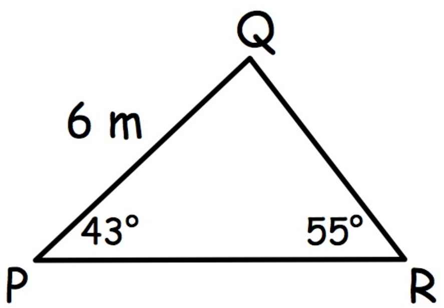


SIDE:

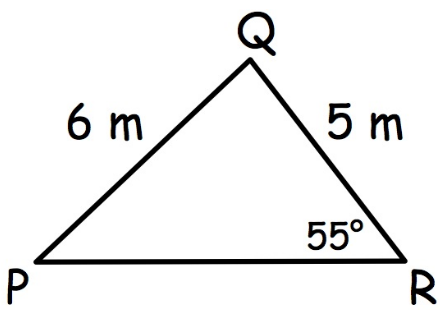
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ANGLE:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Find the length of QR.



Find angle QPR.

$$\frac{QR}{\sin 43°} = \frac{6}{\sin 55°}$$

$$QR = \frac{6}{\sin 55°} \times \sin 43°$$

$$= 4.995\ldots$$

$QR \approx 5.0$ m

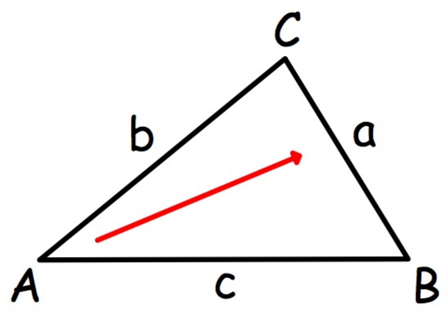
$$\frac{\sin P}{5} = \frac{\sin 55°}{6}$$

$$\sin P = \frac{\sin 55°}{6} \times 5$$

$$\sin P = 0.682\ldots$$

$\angle P \approx 43°$

COSINE RULE



must know two sides and the angle between them

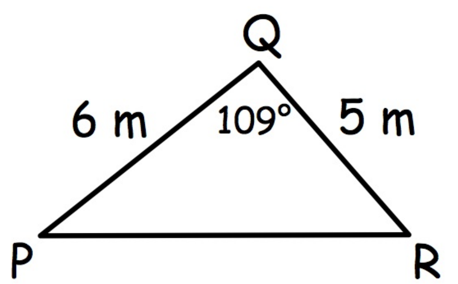
SIDE:

$a^2 = b^2 + c^2 - 2bc \cos A$

must know three sides

ANGLE:

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$



Find the length of PR.

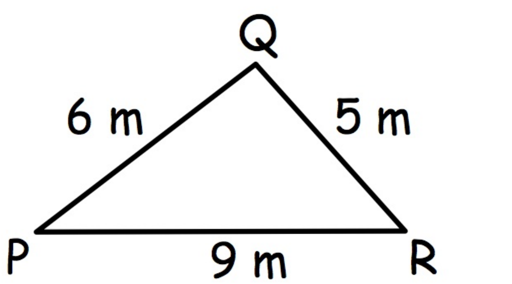
$PR^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 109°$

$= 80.534...$

$PR = \sqrt{80.534...}$

$= 8.974....$

$PR \approx 9.0$ m



Find angle PQR.

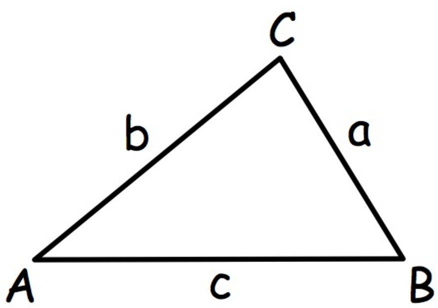
$\cos Q = \dfrac{(6^2 + 5^2 - 9^2)}{(2 \times 6 \times 5)}$

$\cos Q = -0.333...$

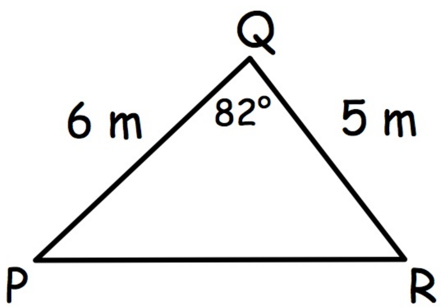
$\angle Q = 109.471...$

$\approx 109°$

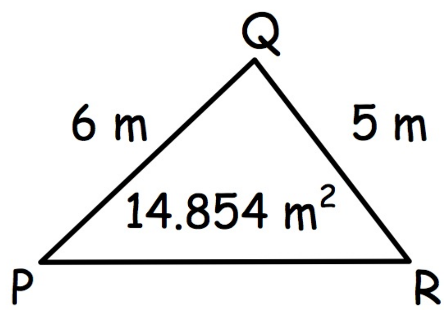
AREA FORMULA must know two sides and the angle between them



$$\text{area } \Delta ABC = \frac{1}{2}ab\sin C$$



Find the area.



Find angle PQR.

$$\begin{aligned} \text{area} &= 0.5 \times 6 \times 5 \times \sin 82^\circ \\ &= 14.854.... \\ &\approx \underline{\underline{14.9 \text{ m}^2}} \end{aligned}$$

$$\begin{aligned} 14.854 &= 0.5 \times 6 \times 5 \times \sin Q \\ 14.854 &= 15 \times \sin Q \\ \sin Q &= 14.854 \div 15 \\ &= 0.990... \end{aligned}$$

$$\underline{\underline{\angle Q \approx 82^\circ \text{ or } 98^\circ}}$$

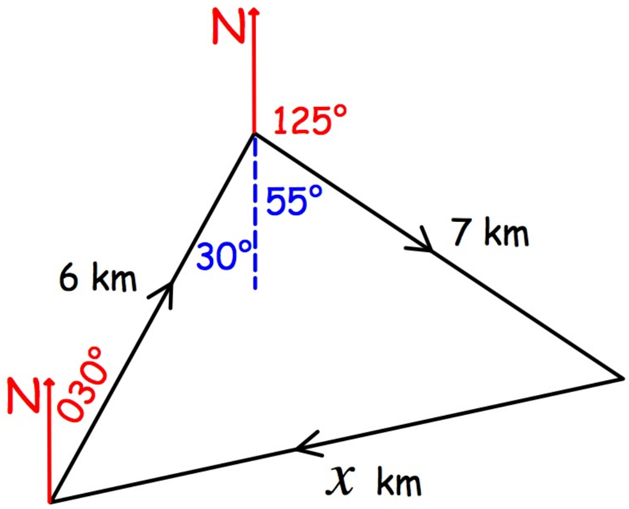
APPLICATIONS

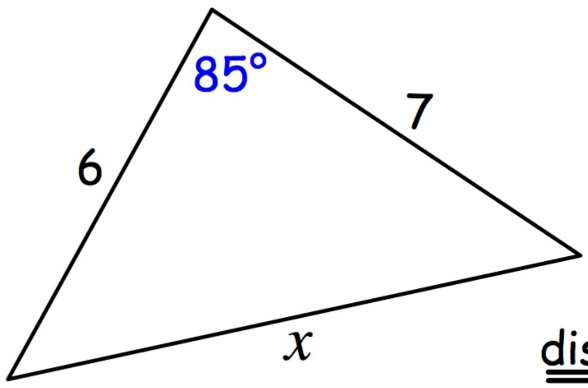
Sketch a diagram and obtain all possible angles.

Sine Rule: know side and opposite angle.
Cosine Rule: know two sides and included angle.

(1) A ship leaves port and travels 6 km on bearing 030°.
It then travels 7 km on bearing 125°.

Find the distance of the ship from port.



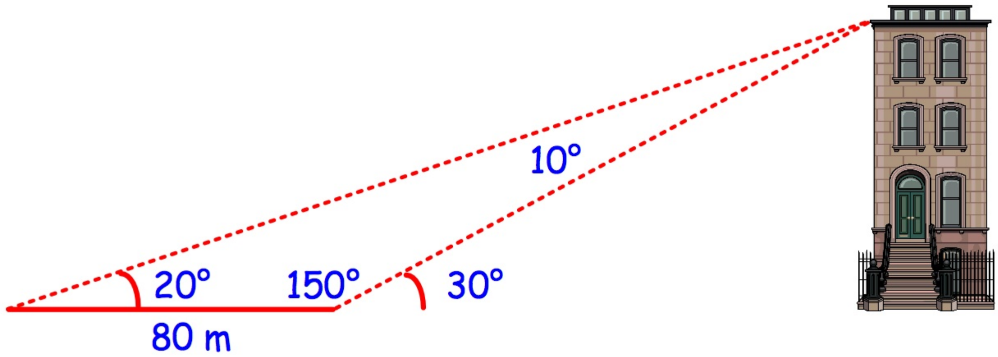


$$x^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 85°$$
$$= 77.678....$$
$$x = \sqrt{77.678....}$$
$$= 8.813....$$

distance 8.8 km

(2) To find the height of a building the angle of elevation is measured from two positions 80m apart.
The angles are 20° and 30°

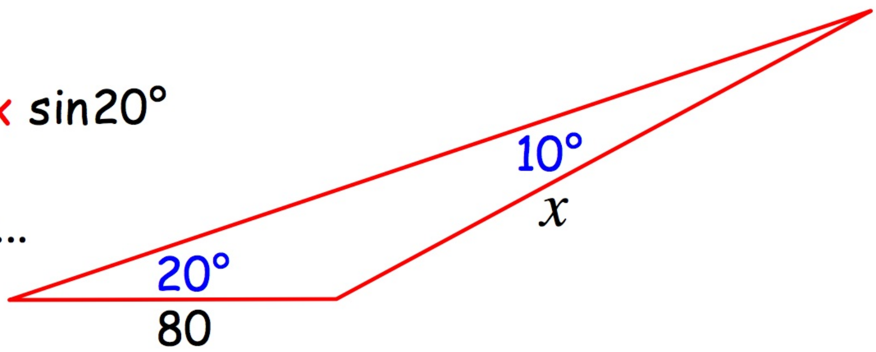
Find the height of the building.



$$\frac{x}{\sin 20°} = \frac{80}{\sin 10°}$$

$$x = \frac{80}{\sin 10°} \times \sin 20°$$

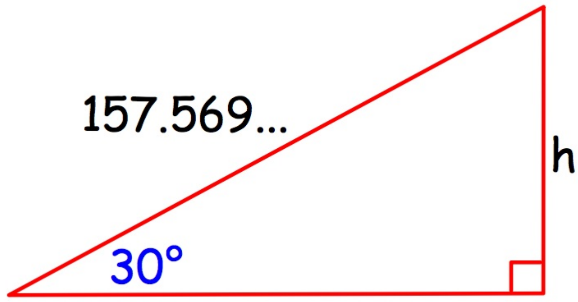
$$= 157.569...$$



$$\sin 30° = \frac{h}{157.569...}$$

$$h = 157.569... \times \sin 30°$$

$$= 78.784...$$



height 78.8 m

VECTORS

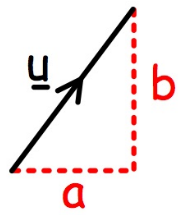
SCALAR quantities have SIZE (magnitude).

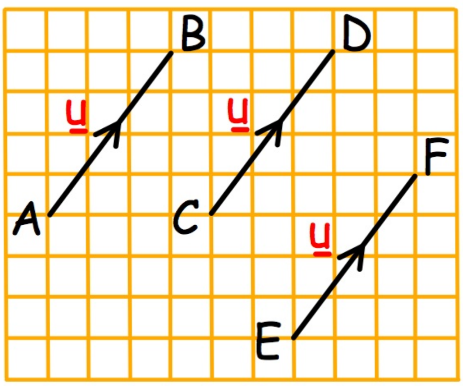
VECTOR quantities have SIZE and DIRECTION.

DIRECTED LINE SEGMENT

A line of a particular size and direction is used to represent a vector.

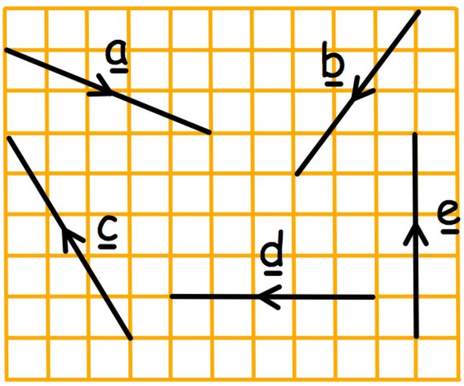
COMPONENT FORM $\underline{u} = \begin{pmatrix} a \\ b \end{pmatrix}$





$\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{EF} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

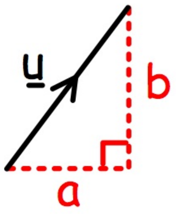
Three directed line segments,
same size and direction,
same component form,
same vector **u**. $\underline{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$



$\underline{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$\underline{c} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $\underline{d} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$ $\underline{e} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

MAGNITUDE   Follows from Pyth. Thm.



$\underline{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $|\underline{u}| = \sqrt{a^2 + b^2}$

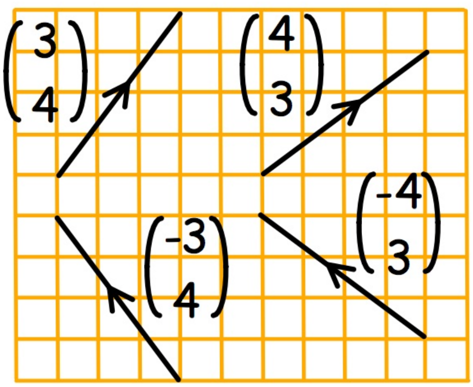
$\vec{AB} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$|\vec{AB}| = \sqrt{(-3)^2 + 6^2}$

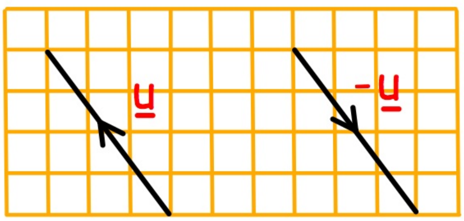
$= \sqrt{45}$

$= \underline{\underline{3\sqrt{5} \text{ units}}}$

NOTE: different vectors can have the same magnitude.



all different vectors
same magnitude 5 units.



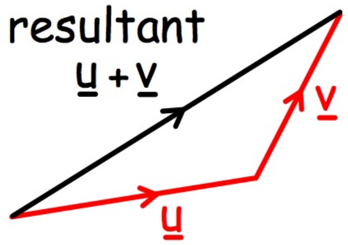
$\begin{pmatrix} 3 \\ -4 \end{pmatrix} = -\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$-\underline{u}$ is the NEGATIVE of $\underline{u}$

The direction is reversed.

ADD and SUBTRACT

By "head-to-tail" triangle.

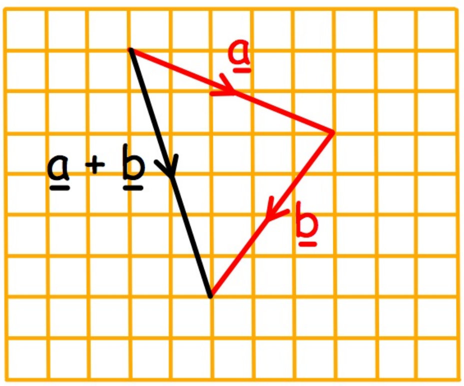


NOTE: $|\underline{u}| + |\underline{v}| > |\underline{u} + \underline{v}|$

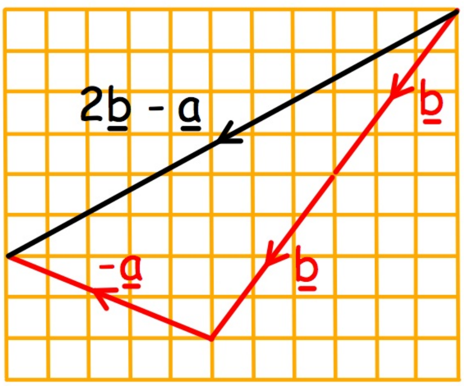
By components: add or subtract components.

MULTIPLY BY A SCALAR: multiply components.

$$k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$



$$\underline{a} + \underline{b} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$



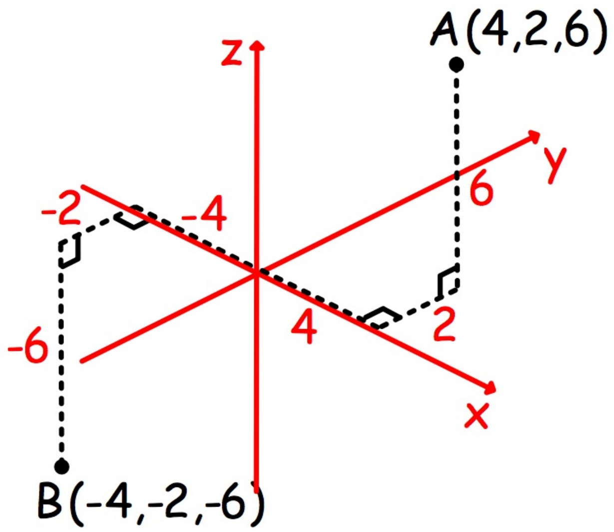
$$2\underline{b} - \underline{a} = 2\begin{pmatrix} -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

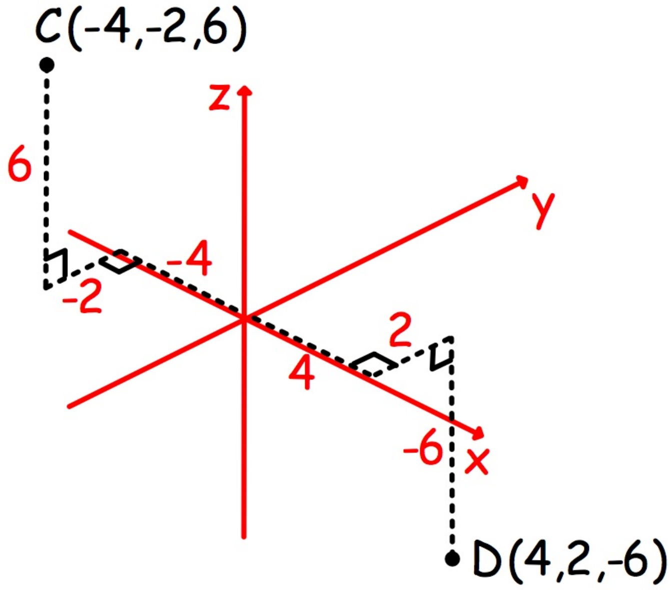
$$= \begin{pmatrix} -6 \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ -6 \end{pmatrix}$$

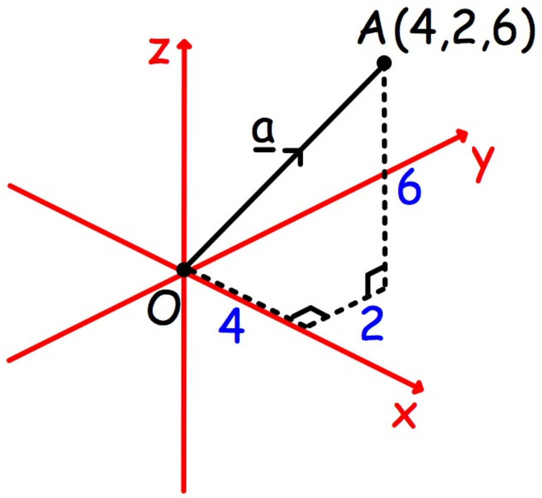
3D COORDINATES

Points (x, y, z) plotted on 3 mutually perpendicular axes.



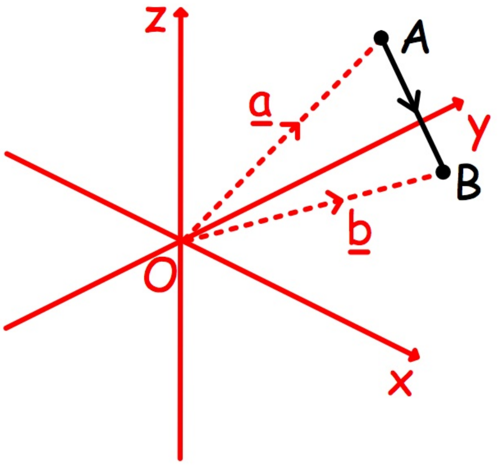


The POSITION VECTOR of point A is given by $\overrightarrow{OA}$.



$$\underline{a} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$$

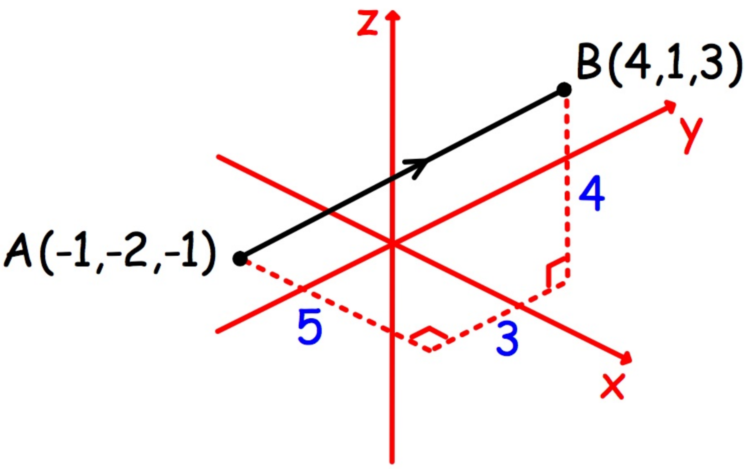
POSITION VECTORS



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

$$= \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$$

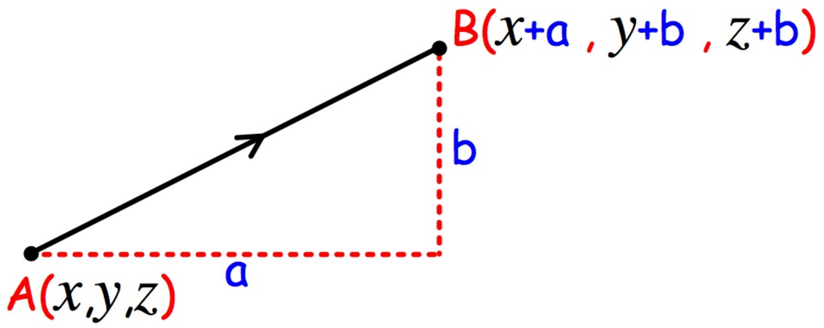
OR

+5 +3 +4

A(-1 , -2 , -1) ⟶ B(4 , 1 , 3)

TRANSLATION $\overrightarrow{AB}$ represents a movement from A to B

$$\overrightarrow{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

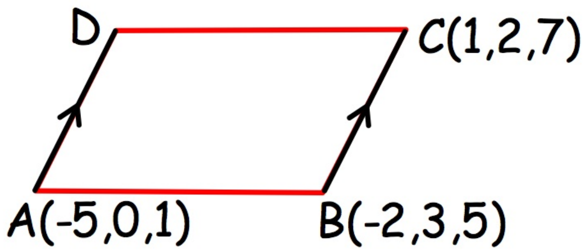


(1) If $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$ and A(-1,-2,-1), find the coordinates of B.

A(-1,-2,-1) ⟶ B(-1 + 5 , -2 + 3 , -1 + 4)

<u>B(4,1,3)</u>

(2) For parallelogram ABCD, find the coordinates of D.



+3 -1 +2

B(-2 , 3 , 5) ⟶ C(1 , 2 , 7)

$$\overrightarrow{BC} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

parallelogram: $\overrightarrow{AD} = \overrightarrow{BC} \Rightarrow \overrightarrow{AD} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

+3 -1 +2

A(-5 , 0 , 1) ⟶ D

<u>D(-2,-1,3)</u>

MAGNITUDE   Follows from Pyth. Thm.

$$\underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad |\underline{u}| = \sqrt{a^2 + b^2 + c^2}$$

Find the distance from A(-2,3,5) to B(1,2,7).

$$\vec{AB} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \qquad |\vec{AB}| = \sqrt{3^2 + (-1)^2 + 2^2}$$

$$= \underline{\underline{\sqrt{14} \text{ units}}}$$

ADD and SUBTRACT: add or subtract components.

MULTIPLY BY A SCALAR: $k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$

If $\underline{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$, find $|\underline{b} - 2\underline{a}|$.

$$\underline{b} - 2\underline{a} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$$

$$|\underline{b} - 2\underline{a}| = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \underline{\underline{\sqrt{41} \text{ units}}}$$